

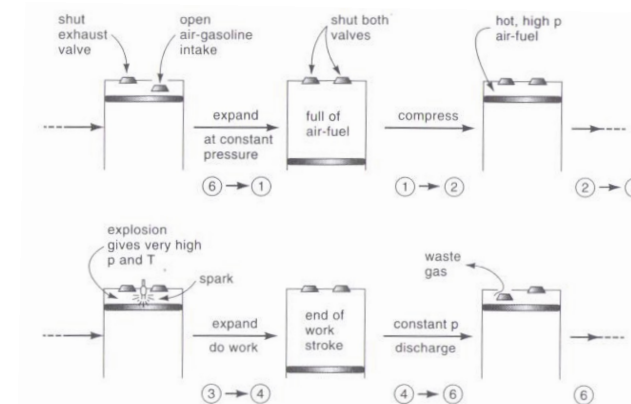
## Outline of Part 1: Thermo

**Objective:** Derive and understand the physical laws that characterize and limit energy conversion systems

- 1<sup>st</sup> Law of Thermodynamics
  - Internal energy
  - Work
  - Enthalpy
- 2<sup>nd</sup> Law of Thermodynamics
  - Entropy
  - Reversible and irreversible processes
  - State functions
- Heat to work conversion
  - T-S diagrams
  - Idealized systems (Carnot cycle)
- Real heat to work and work to heat conversion systems
  - Rankine cycles
  - Refrigeration cycles and heat pumps
  - **Engines**
- Exergy: calculating the maximum work that can be produced/recovered
- Electrical systems
  - Electrical machines
  - Fuel cells

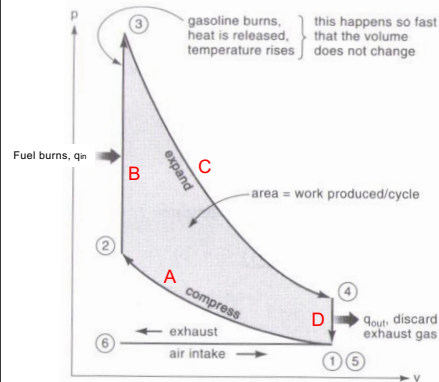
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## Gasoline engines: the Otto cycle



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## Gasoline engines: the Otto cycle

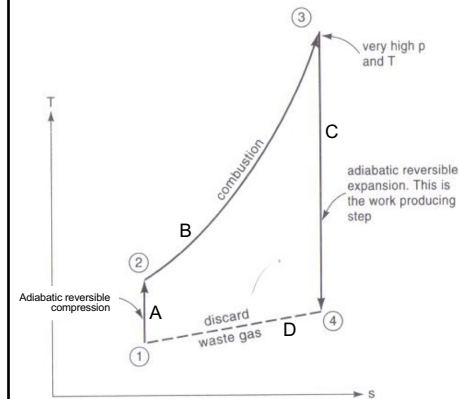


### The cycle:

- A. Adiabatic compression of the fuel mixture
- B. Constant volume heating of the fuel during explosion
- C. Adiabatic expansion of the hot gas (produces work)
- D. Exhaust exits with residual heat at constant volume (removes heat)

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## Gasoline engines: the Otto cycle



### The cycle:

- A. Adiabatic compression  
 $Q = 0$  and  $\Delta U_{1 \rightarrow 2} = W_{in} = C_V(T_2 - T_1)$
- B. Constant volume heating  
 $W = 0$  and  $Q_{in} = C_V(T_3 - T_2)$
- C. Adiabatic expansion  
 $Q = 0$  and  $\Delta U_{3 \rightarrow 4} = W_{out} = C_V(T_3 - T_4)$
- D. Constant volume cooling  
 $W = 0$  and  $Q_{out} = C_V(T_4 - T_1)$

$$\eta = \frac{W}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = \frac{C_V(T_3 - T_2) - C_V(T_4 - T_1)}{C_V(T_3 - T_2)}$$

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## Gasoline engines: the Otto cycle

$$\eta = \frac{C_V(T_3 - T_2) - C_V(T_4 - T_1)}{C_V(T_3 - T_2)}$$

$$= \frac{(T_3 - T_2) - (T_4 - T_1)}{(T_3 - T_2)}$$

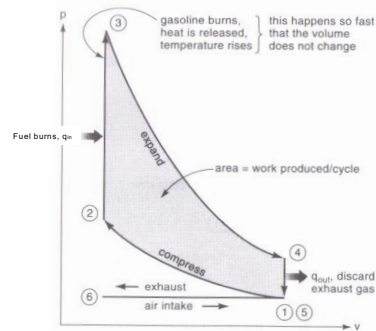
For adiabatic changes we can write:

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} \quad \left(\frac{V_4}{V_3}\right)^{k-1} = \frac{T_3}{T_4}$$

Notice that  $V_1 = V_4$  and  $V_2 = V_3$ .

$$\eta = \frac{(1 - T_2/T_3) - (T_4/T_3 - T_1/T_3)}{(1 - T_2/T_3)} = \frac{(1 - T_2/T_3) - (T_1/T_3 - T_1/T_3)}{(1 - T_2/T_3)} = 1 - \frac{(T_1/T_3 - T_1/T_3)}{(1 - T_2/T_3)} = 1 - T_1/T_2 \frac{(1 - T_2/T_3)}{(1 - T_2/T_3)} = 1 - T_1/T_2$$

$$\eta = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{V_2}{V_1}\right)^{1-k} = 1 - r_c^{1-k}$$



## Gasoline engines: the Otto cycle

$$\eta = 1 - \left(\frac{V_2}{V_1}\right)^{1-k} = 1 - r_c^{1-k}$$

The compression ratio:

$$r_c = \frac{V_1}{V_2} = \frac{V_4}{V_3}$$

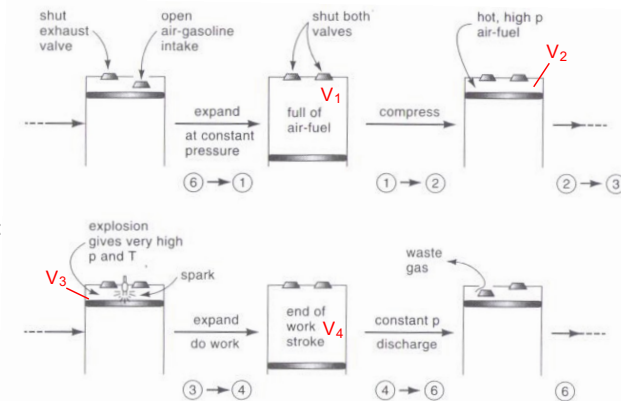
In gasoline engines:

$$r_c \approx 8-9$$

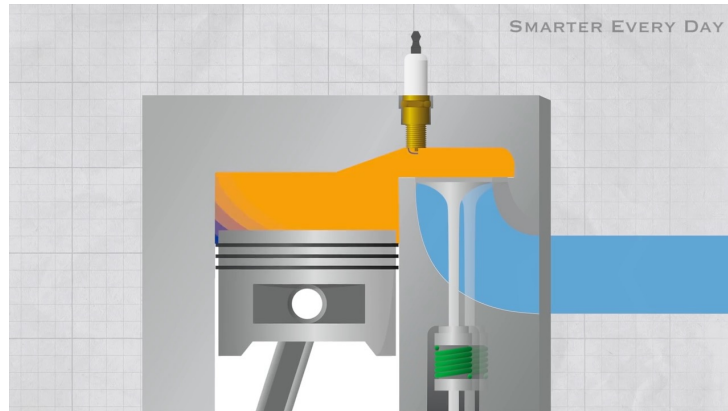
$$\eta \approx 45\%$$

In a real engine:

$$\eta \approx 20-35\%$$

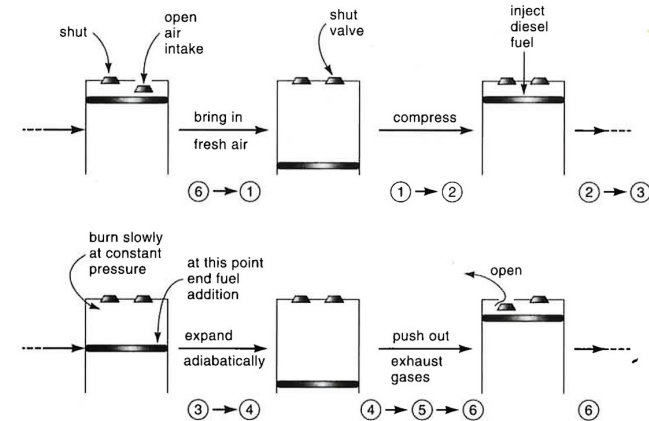


## Gasoline engines



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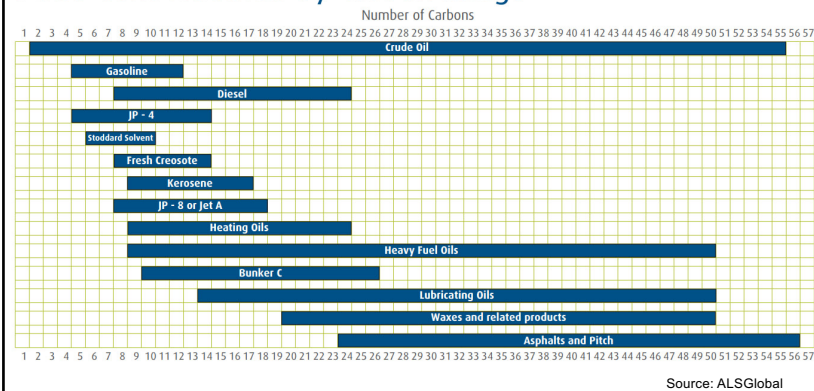
## Diesel engines: the Diesel cycle



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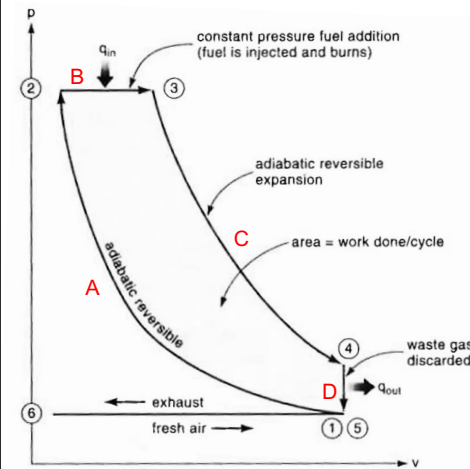
## Diesel vs. Gasoline

### Petroleum Fractions by Carbon Range



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## Diesel engines: the Diesel cycle



### The cycle:

A. Adiabatic compression of air

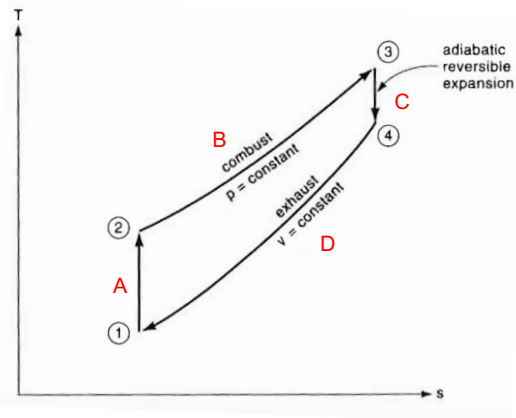
B. Slow fuel injection creates a slow burn/heating of the air at constant pressure leading to an isobaric expansion (produces work)

C. Continued (adiabatic) expansion of the hot gas (produces work)

D. Exhaust exits with residual heat at constant volume (removes heat)

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## Diesel engines: the Diesel cycle

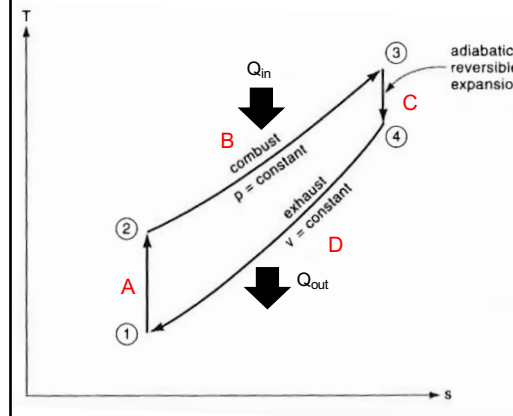


### The cycle:

- A. Adiabatic compression of air
- B. Slow fuel injection creates a slow burn/heating of the air at constant pressure leading to an isobaric expansion (produces work)
- C. Continued (adiabatic) expansion of the hot gas (produces work)
- D. Exhaust exits with residual heat at constant volume (removes heat)

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## Diesel engines: the Diesel cycle

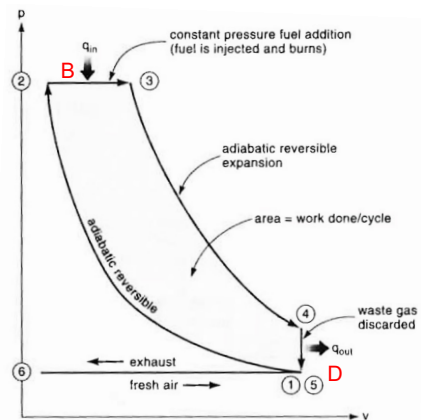


### Key steps:

$$\eta = \frac{W}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = \frac{|Q_{2-3}| - |Q_{4-5}|}{|Q_{2-3}|}$$

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## Diesel engines: the Diesel cycle



### Key steps:

B. Constant pressure heating

$$Q_{in} = \Delta U + P\Delta V = \Delta H = C_p(T_3 - T_2)$$

D. Constant volume cooling  
 $W = 0$  and  $Q_{out} = C_v(T_4 - T_1)$

$$\eta = \frac{W}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = \frac{|Q_{2-3}| - |Q_{4-1}|}{|Q_{2-3}|} = \frac{C_p(T_3 - T_2) - C_v(T_4 - T_1)}{C_p(T_3 - T_2)}$$

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## Diesel engines: the Diesel cycle

$$\eta = \frac{W}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = \frac{C_p(T_3 - T_2) - C_v(T_4 - T_1)}{C_p(T_3 - T_2)}$$

$$= 1 - \frac{C_v(T_4 - T_1)}{C_p(T_3 - T_2)} = 1 - \frac{1}{k} \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

We can define:

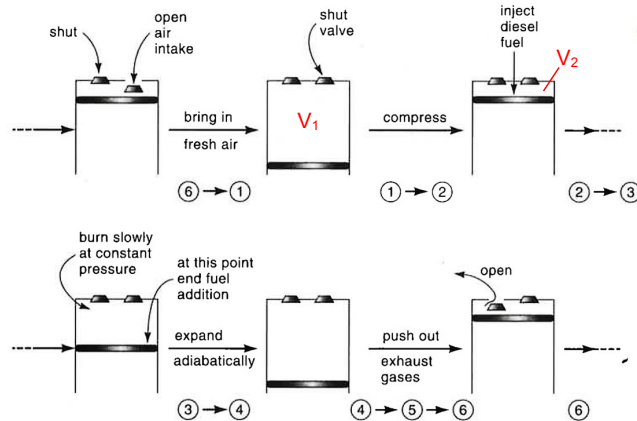
The compression ratio:

$$r_c = \frac{V_1}{V_2}$$

Same as for gasoline engines...

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## Diesel engines: the Diesel cycle



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## Diesel engines: the Diesel cycle

$$\eta = \frac{W}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = \frac{C_p(T_3 - T_2) - C_v(T_4 - T_1)}{C_p(T_3 - T_2)}$$

$$= 1 - \frac{C_v(T_4 - T_1)}{C_p(T_3 - T_2)} = 1 - \frac{1}{k} \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

We can define:

The compression ratio:

$$r_c = \frac{V_1}{V_2}$$

Same as for gasoline engines...

The expansion ratio:

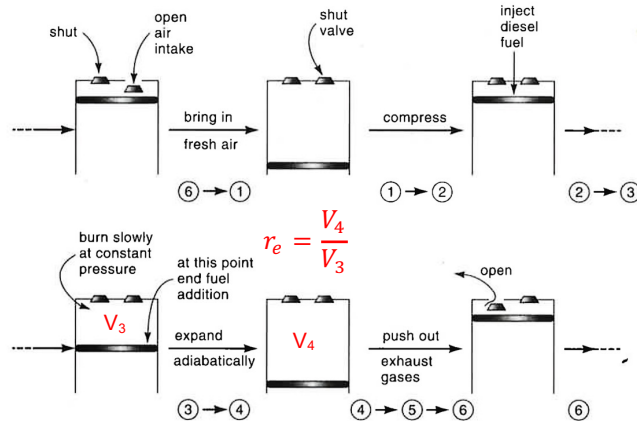
$$r_e = \frac{V_4}{V_3}$$

Defines the further expansion after the injection of fuel

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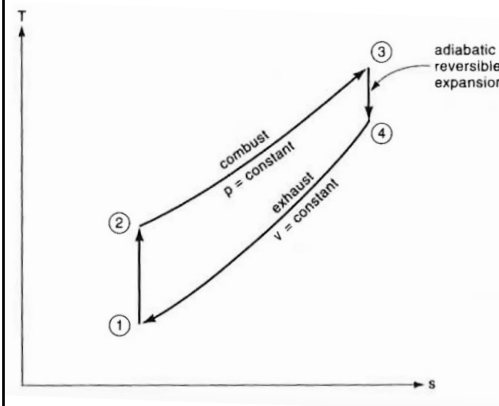
## Diesel engines: the Diesel cycle



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## Diesel engines: the Diesel cycle

Steps 1-2 and 3-4 are adiabatic...



The compression ratio:

$$r_c = \frac{V_1}{V_2} = \left( \frac{T_2}{T_1} \right)^{1/(k-1)}$$

$$T_2 = T_1 (r_c)^{k-1}$$

$$\rightarrow T_1 = T_2 \left( \frac{1}{r_c} \right)^{k-1}$$

The expansion ratio:

$$r_e = \frac{V_4}{V_3} = \left( \frac{T_3}{T_4} \right)^{1/(k-1)}$$

$$T_3 = T_4 (r_e)^{k-1}$$

$$\rightarrow T_4 = T_3 \left( \frac{1}{r_e} \right)^{k-1}$$

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## Diesel engines: the Diesel cycle

$$\eta = 1 - \frac{1}{k} \frac{(T_4 - T_1)}{(T_3 - T_2)} \quad T_1 = T_2 \left( \frac{1}{r_c} \right)^{k-1} \quad T_4 = T_3 \left( \frac{1}{r_e} \right)^{k-1}$$

Let's further define:

$$\begin{aligned} \frac{(T_4 - T_1)}{(T_3 - T_2)} &= \frac{T_3 \left( \frac{1}{r_e} \right)^{k-1} - T_2 \left( \frac{1}{r_c} \right)^{k-1}}{(T_3 - T_2)} = \frac{\frac{T_3(T_3 - T_2)}{T_3 - T_2} \left( \frac{1}{r_e} \right)^{k-1} - \frac{T_2(T_3 - T_2)}{T_3 - T_2} \left( \frac{1}{r_c} \right)^{k-1}}{(T_3 - T_2)} \\ &= \frac{\frac{(T_3 - T_2)}{1 - \frac{T_2}{T_3}} \left( \frac{1}{r_e} \right)^{k-1} - \frac{(T_3 - T_2)}{\frac{T_3}{T_2} - 1} \left( \frac{1}{r_c} \right)^{k-1}}{(T_3 - T_2)} = \frac{(T_3 - T_2) \left( \frac{\left( \frac{1}{r_e} \right)^{k-1}}{1 - T_2/T_3} - \frac{\left( \frac{1}{r_c} \right)^{k-1}}{T_3/T_2 - 1} \right)}{(T_3 - T_2)} \\ &= \left( \frac{\left( \frac{1}{r_e} \right)^{k-1}}{1 - T_2/T_3} - \frac{\left( \frac{1}{r_c} \right)^{k-1}}{T_3/T_2 - 1} \right) \end{aligned}$$

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## Diesel engines: the Diesel cycle

$$\eta = 1 - \frac{1}{k} \left( \frac{\left( \frac{1}{r_e} \right)^{k-1}}{1 - T_2/T_3} - \frac{\left( \frac{1}{r_c} \right)^{k-1}}{T_3/T_2 - 1} \right)$$

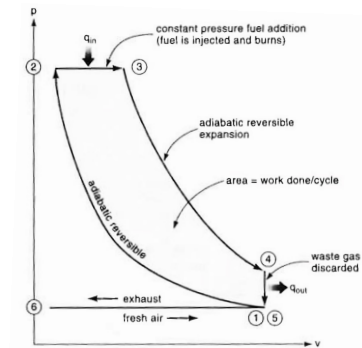
Step 2→3 is isobaric  
 $P_2 = P_3$

Step 4→1 is isochoric  
 $V_4 = V_1$

With the ideal gas law:

$$\begin{aligned} \frac{T_3}{T_2} &= \frac{P_3 V_3}{P_2 V_2} \\ &= \frac{V_3}{V_2} = \frac{V_3/V_4}{V_2/V_1} = \frac{r_c}{r_e} \end{aligned}$$

$$\eta = 1 - \frac{1}{k} \left( \frac{\left( \frac{1}{r_e} \right)^{k-1}}{1 - r_e/r_c} - \frac{\left( \frac{1}{r_c} \right)^{k-1}}{r_c/r_e - 1} \right)$$



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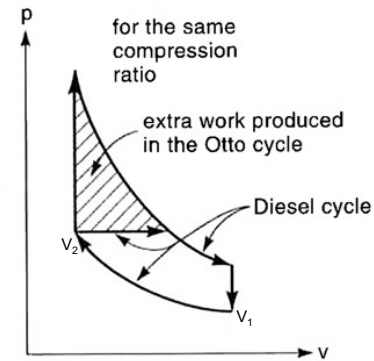
## Diesel engines: the Diesel cycle

$$\eta = 1 - \frac{1}{k} \left( \frac{\left(\frac{1}{r_e}\right)^{k-1}}{1 - r_e/r_c} - \frac{\left(\frac{1}{r_c}\right)^{k-1}}{r_c/r_e - 1} \right)$$

$$= 1 - \frac{1}{k} \left( \frac{\left(\frac{1}{r_e}\right)^k}{1/r_e - 1/r_c} - \frac{\left(\frac{1}{r_c}\right)^k}{1/r_e - 1/r_c} \right)$$

$$= 1 - \frac{1}{k} \frac{\left(\frac{1}{r_e}\right)^k - \left(\frac{1}{r_c}\right)^k}{1/r_e - 1/r_c}$$

For the same compression ratio,  
 $r_c = \frac{V_1}{V_2}$  the Otto cycle is more efficient.



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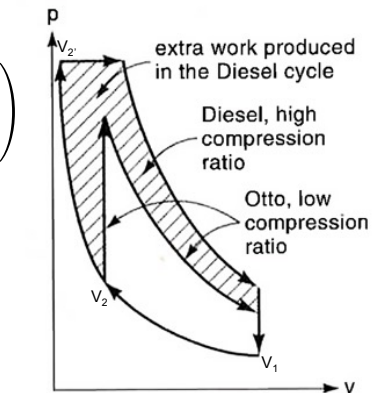
## Diesel engines: the Diesel cycle

$$\eta = 1 - \frac{1}{k} \left( \frac{\left(\frac{1}{r_e}\right)^{k-1}}{1 - r_e/r_c} - \frac{\left(\frac{1}{r_c}\right)^{k-1}}{r_c/r_e - 1} \right)$$

$$= 1 - \frac{1}{k} \left( \frac{\left(\frac{1}{r_e}\right)^k}{1/r_e - 1/r_c} - \frac{\left(\frac{1}{r_c}\right)^k}{1/r_e - 1/r_c} \right)$$

$$= 1 - \frac{1}{k} \frac{\left(\frac{1}{r_e}\right)^k - \left(\frac{1}{r_c}\right)^k}{1/r_e - 1/r_c}$$

But in practice, higher compression ratios can be achieved (20 vs. 8-9 for gasoline engines) → This is because only air is compressed instead of a gasoline fuel mixture.



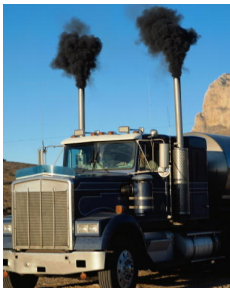
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## Diesel engines: the Diesel cycle

The higher compression ratios, lead to theoretical efficiencies close to 60%.

In real engines, efficiencies can reach a little over 40%, making them more fuel efficient than gasoline engines.

Lack of pre-mixing causes gradients with uneven combustion:

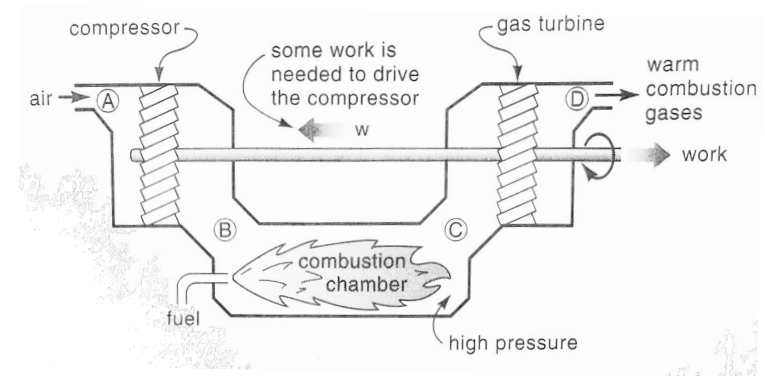


Higher compression = higher temperature = higher NOx formation



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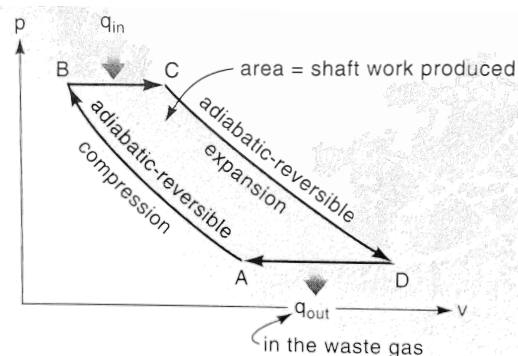
## Gas turbines and jet engines: the Brayton cycle



Advantage over the Rankine cycle: no need for a heat exchange network.

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## Gas turbines and jet engines: the Brayton cycle



### The cycle:

A-B. Air enters the turbine and is compressed adiabatically.

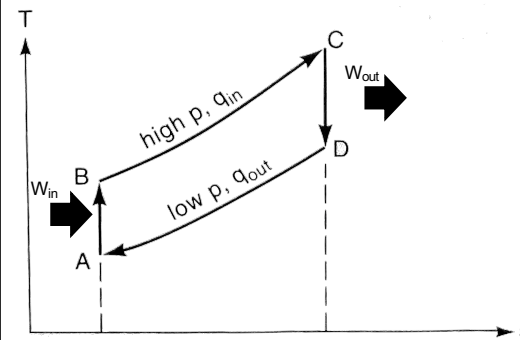
B-C. Compressed air is mixed with burning fuel: heating occurs at constant pressure and volume expands

C-D. Hot gas is expanded back to atmospheric pressure adiabatically and produces work

D-A. Exhaust exits and cools back down at constant pressure (and contracts in doing so).

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## Gas turbines and jet engines: the Brayton cycle



### The cycle:

A-B. Air enters the turbine and is compressed adiabatically.

B-C. Compressed air is mixed with burning fuel: heating occurs at constant pressure and volume expands

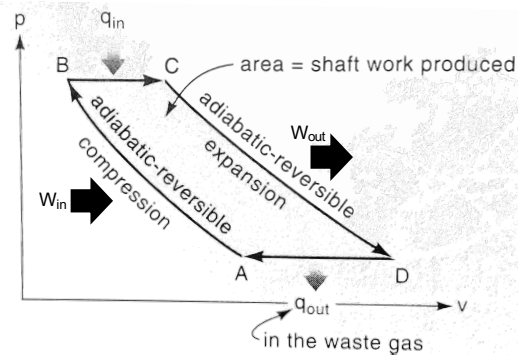
C-D. Hot gas is expanded back to atmospheric pressure adiabatically and produces work

D-A. Exhaust exits and cools back down at constant pressure (and contracts in doing so).

$$\eta = \frac{\text{Excess work}}{\text{Heat released by the fuel}} = \frac{|W_{CD}| - |W_{AB}|}{|Q_{BC}|}$$

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## Gas turbines and jet engines: the Brayton cycle



### Key steps:

B-C. Constant pressure heating:

$$\begin{aligned} Q &= \Delta U + P\Delta V \\ &= \Delta H \\ &= C_p(T_C - T_B) \end{aligned}$$

D-A. Constant pressure cooling:

$$\begin{aligned} Q &= \Delta U + P\Delta V \\ &= \Delta H \\ &= C_p(T_D - T_A) \end{aligned}$$

By energy balance:

$$\eta = \frac{|W_{CD}| - |W_{AB}|}{|Q_{BC}|} = \frac{|Q_{BC}| - |Q_{AD}|}{|Q_{BC}|} = \frac{C_p(T_C - T_B) - C_p(T_D - T_A)}{C_p(T_C - T_B)}$$

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## Gas turbines and jet engines: the Brayton cycle

$$\eta = \frac{(T_C - T_B) - (T_D - T_A)}{(T_C - T_B)}$$

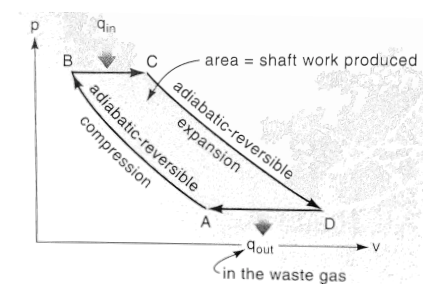
Since A-B and C-D are adiabatic:

$$\left(\frac{T_B}{T_A}\right) = \left(\frac{P_B}{P_A}\right)^{\frac{k-1}{k}} \quad \left(\frac{T_C}{T_D}\right) = \left(\frac{P_C}{P_D}\right)^{\frac{k-1}{k}}$$

$$\text{And } P_B = P_C \quad P_A = P_D$$

$$\left(\frac{T_B}{T_A}\right) = \left(\frac{P_B}{P_A}\right)^{\frac{k-1}{k}} = \left(\frac{P_C}{P_D}\right)^{\frac{k-1}{k}} = \left(\frac{T_C}{T_D}\right)$$

$$\eta = 1 - \frac{(T_D - T_A)}{(T_C - T_B)} = 1 - \frac{T_A}{T_B} \frac{\left(\frac{T_D}{T_A} - 1\right)}{\left(\frac{T_C}{T_B} - 1\right)} = 1 - \frac{T_A}{T_B} \frac{\left(\frac{T_C}{T_B} - 1\right)}{\left(\frac{T_C}{T_B} - 1\right)} = 1 - \frac{T_A}{T_B} = 1 - \left(\frac{P_A}{P_B}\right)^{\frac{k-1}{k}}$$



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## Gas turbines and jet engines: the Brayton cycle

$$\eta = 1 - \left( \frac{P_A}{P_B} \right)^{\frac{k-1}{k}} \longrightarrow \text{This means that the higher the compression, the higher the efficiency.}$$

Higher compression ratios also leads to higher post combustion temperatures, which is limited by materials (material limits are between 1200-1500°C).

Practical limits of a gas turbine lead to pressure ratios around 20 and theoretical efficiencies around 55-60%.

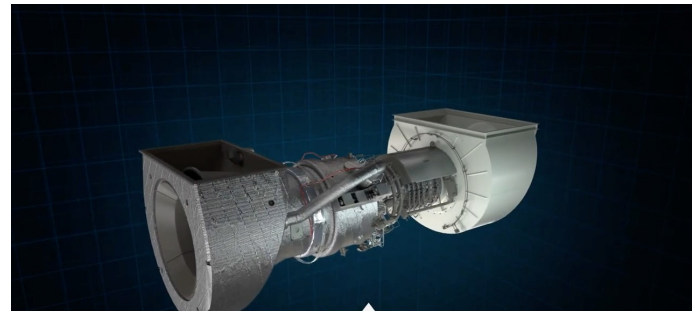
The compressor employs a significant amount of work (about 80% of the output work\* is employed by the compressor). And compressors are never more than 80% efficient.

Real world efficiencies can now reach about 40-45%.

\*which is not the net work, but the work done by the output turbine...

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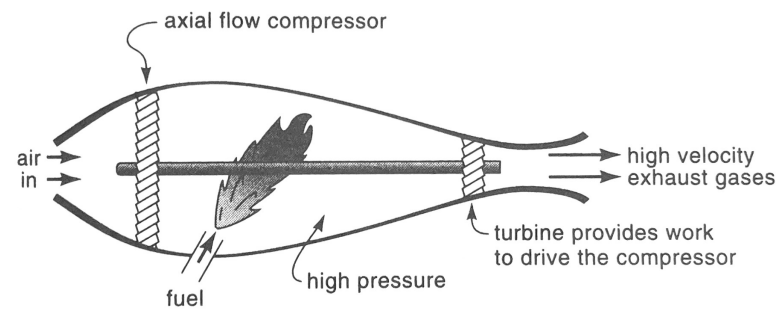
## A gas turbine



<https://www.youtube.com/watch?v=irWHi77oAWI>

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## Brayton cycles applied to jet engines



For a jet engine: same general idea but a different geometry that is built to accelerate the exiting gas.

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## Aircraft evolution



Boeing 747-100 (1966)



Boeing 777X (2019)

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## Brayton cycles applied to jet engines

Ultimately, for a jet engine we care about *propulsive power* and the amount produced with respect to the thermal energy of the fluid.

$$\eta_{overall} = \frac{\text{propulsive power}}{\dot{Q}_{in}}$$

It is useful to break this efficiency down into two parts:

$$\eta_{overall} = \eta_{thermal} \eta_{propulsive}$$

$\eta_{thermal}$  Represents the ratio of kinetic energy created over the thermal energy given by the fuel.

If we neglect the mass of fuel added to the air:  $\dot{m}_{air,in} = \dot{m}_{air,out} = \dot{m}_{air}$

$$\eta_{thermal} = \frac{(m_{air,out} v_{out}^2 - m_{air,in} v_{in}^2)/2}{\dot{Q}_{in}} = 1 - \frac{T_A}{T_B}$$

Because all of the pV work is used to accelerate the gas.

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## Brayton cycles applied to jet engines

Ultimately, for a jet engine we care about *propulsive power* and the amount produced with respect to the thermal energy of the fluid.

$$\eta_{overall} = \frac{\text{propulsive power}}{\dot{Q}_{in}}$$

It is useful to break this efficiency down into two parts:

$$\eta_{overall} = \eta_{thermal} \eta_{propulsive}$$

$\eta_{propulsive}$  The ratio of propulsive power produced, divided by the rate of production of the kinetic energy transferred to the gas.

$$\eta_{propulsive} = \frac{\text{propulsive power}}{\text{rate of production of propulsive kinetic energy}}$$

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## Brayton cycles applied to jet engines

$$\eta_{propulsive} = \frac{\text{propulsive power}}{\text{rate of production of propulsive kinetic energy}}$$

Again, neglecting the mass of fuel, we can just use the mass of air:

$$\eta_{propulsive} = \frac{\text{propulsive power}}{\dot{m}_{air} (v_{out}^2 - v_{in}^2)/2}$$

Flight speed, which is equivalent to the speed incoming air  $v_{in}$

$$\text{propulsive power} = v_{flight} \times \text{Thrust}$$

$$= v_{in} (\dot{m}_{air} (v_{out} - v_{in}))$$

$$\eta_{propulsive} = \frac{v_{in} (\dot{m}_{air} (v_{out} - v_{in}))}{\dot{m}_{air} (v_{out}^2 - v_{in}^2)/2} = \frac{2 v_{in} (v_{out} - v_{in})}{(v_{out} + v_{in})(v_{out} - v_{in})} = \frac{2 v_{in}}{v_{out} + v_{in}} = \frac{2}{1 + \frac{v_{out}}{v_{in}}}$$

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## Brayton cycles applied to jet engines

$$\eta_{propulsive} = \frac{2}{1 + \frac{v_{out}}{v_{in}}}$$

The propulsive efficiency only depends on speed and is the highest when the increase in speed is small.

But the airplane must maintain a high degree of thrust to:

- Counter air resistance (which is ignored in ideal calculations)
- Maintain the ability to accelerate

$$\text{thrust} = \dot{m}_{air} (v_{out} - v_{in})$$

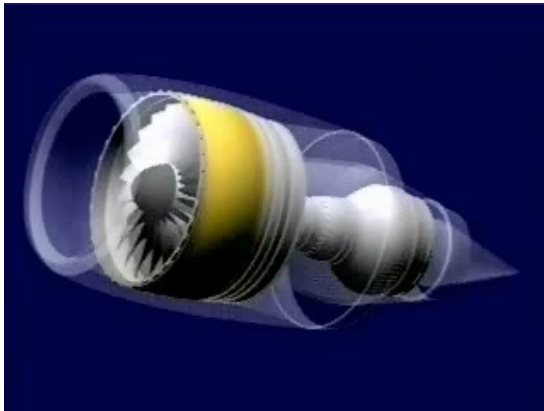
Thrust decreases when air speed difference decreases.

The opposite of efficiency!

However, *thrust* increases with  $\dot{m}_{air}$  → The solution was to increase airflow while keeping speed differences small using so-called high bypass engines

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## Modern engines are high bypass engines

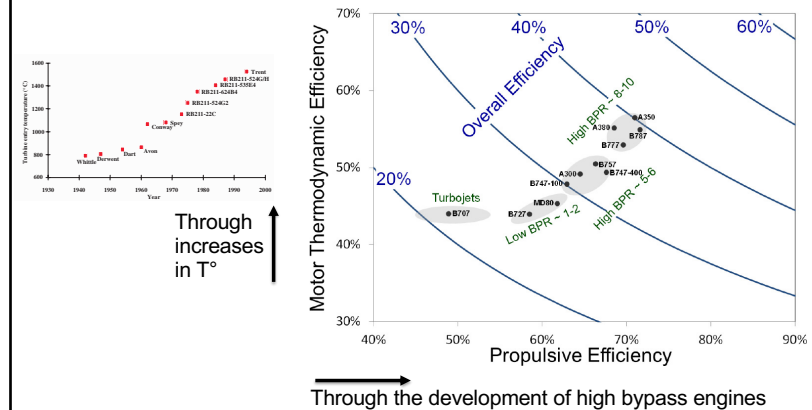


Source: USAF academy

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## Modern engines are high bypass engines

Recent advances have allowed major improvements in both thermodynamic and propulsive efficiency:



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## Outline of Part 1: Thermo

**Objective:** Derive and understand the physical laws that characterize and limit energy conversion systems

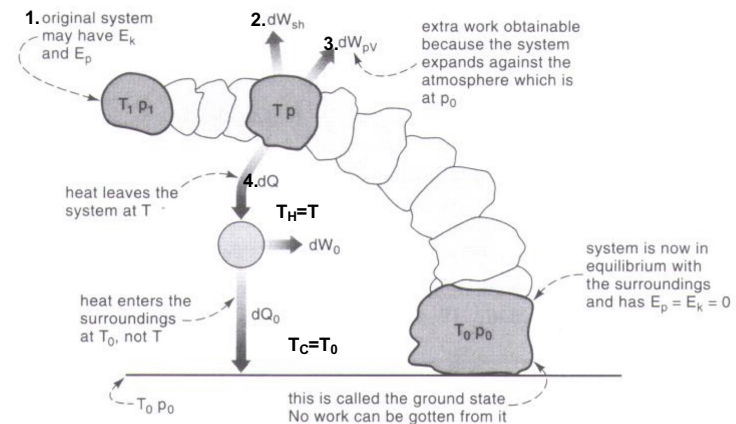
- 1<sup>st</sup> Law of Thermodynamics
  - Internal energy
  - Work
  - Enthalpy
- 2<sup>nd</sup> Law of Thermodynamics
  - Entropy
  - Reversible and irreversible processes
  - State functions
- Heat to work conversion
  - T-S diagrams
  - Idealized systems (Carnot cycle)
- Real heat to work and work to heat conversion systems
  - Rankine cycles
  - Refrigeration cycles and heat pumps
  - Engines
- **Exergy: calculating the maximum work that can be produced/recovered**
- Electrical systems
  - Electrical machines
  - Fuel cells

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## Exergy

**Exergy ( $W_{ex}$ ):** the maximum amount of **useful** work obtainable for a given process.

Let's start with the ground state: All energy has to be accounted for as we bring our system to the ground state. This includes:



Is this just a Carnot Cycle? **No!**  $E_p$ ,  $E_k$  and  $dW_{sh}$  are included as well.

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## Calculating ground state Exergy

1<sup>st</sup> Law:

$$dE = dQ_0 - dW$$

Includes  
 $dU$ ,  $dE_p$   
and  $dE_k$

Is the heat  
released to  
ground state

Includes all  
 $dW = dW_{sh} +$   
 $dW_{pV} + dW_0$

We know that the heat released by a  
Carnot engine is reversible and is at  $T_0$ :

$$dQ_0 = T_0 dS$$

$$dE = T_0 dS - dW_{sh} - dW_{pV} - dW_0 = T_0 dS - dW_{sh} - p_0 dV - dW_0$$

$$dW_{Ex} = dW_{sh} + dW_0 = -dE + T_0 dS - p_0 dV$$

We integrate (with  $E_0 = U_0$ ):

$$W_{Ex,1 \rightarrow 0} = -(U_0 - E_1) + T_0(S_0 - S_1) - p_0(V_0 - V_1)$$

